**6.7**  **The Binomial Random Variable**

There are many situations that have only two outcomes. For example:

* Flip a coin. The outcome is a head or a tail.
* Select a person. The gender is male or female.
* Roll a fair die. You either roll a 6 or you don’t roll a six.
* Take a foul shot. You will either hit or miss the shot.

A **binomial experiment** occurs when all of the following criteria are met:

1. The experiment consists of a fixed number n of **trials**.
2. Each trial results in one of **two outcomes**. We refer to the outcomes as **success** and **failure**.
3. The probability of success for each trial is a **constant value** **p**. The probability of failure for each trial is **1 – p**.
4. The outcome of each trial is **independent** of the outcome of the other trials.

When the random variable X is defined to be the number of successes observed in the n trials, the X is called a **binomial random variable**.

* The parameter **p** is called the **binomial (or population) proportion**. We say that the random variable X has a binomial distribution with parameters n and p.
* The probability distribution function of X is given by p(x) = P(X = x) = nCx⋅px⋅(1 – p)n-x where .
* The numbers nCx are called the **binomial coefficients** and can be found using **Pascal’s triangle**.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | | 1 | | |  | | |  | | |  | | |  | | |  | | |  | | |  | | |  | | | 0C0 | | |  | | |  | | |  | | |  | | |
|  |  |  | | |  | | | 1 | | | 1 | | |  | | |  | | |  | | | |  | | |  | | |  | | |  | | | 1C0 | | | 1C1 | | |  | | |  | | |  | | |
|  |  | | |  | | | 1 | | | 2 | | | 1 | | |  | | |  | | |  | | |  | | |  | | |  | | | 2C0 | | | 2C1 | | | 2C2 | | |  | | |  | | |  | | |
|  |  |  | | | 1 | | | 3 | | | 3 | | | 1 | | |  | | |  | | | |  | | |  | | |  | | | 3C0 | | | 3C1 | | | 3C2 | | | 3C3 | | |  | | |  | | |
|  |  | | | 1 | | | 4 | | | 6 | | | 4 | | | 1 | | |  | | |  | | |  | | |  | | | 4C0 | | | 4C1 | | | 4C2 | | | 4C3 | | | 4C4 | | |  | | |  | | |
|  |  | 1 | | | 5 | | | 10 | | | 10 | | | 5 | | | 1 | | |  | | | |  | | |  | | | 5C0 | | | 5C1 | | | 5C2 | | | 5C3 | | | 5C4 | | | 5C5 | | |  | | |
|  | 1 | | | 6 | | | 15 | | | 20 | | | 15 | | | 6 | | | 1 | | |  | | |  | | | 6C0 | | | 6C1 | | | 6C2 | | | 6C3 | | | 6C4 | | | 6C5 | | | 6C6 | | |  | | |
| 1 | | | 7 | | | 21 | | | 35 | | | 35 | | | 21 | | | 7 | | | 1 | |  | | | 7C0 | | | 7C1 | | | 7C2 | | | 7C3 | | | 7C4 | | | 7C5 | | | 7C6 | | | 7C7 | | |

* It can be shown that the mean of a binomial random variable is  = np.
* It can also be shown that the variance is σ2 = np(1 – p) and the standard deviation is σ = .

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**Example 17**

On average, a basketball player makes 80% of her free throw shots. Consider the player taking 5 free throw shots and assume that outcome of each shot is independent of the outcome of any of the other shots.

1. Examine the four conditions for a binomial experiment.

* The experiment consists of n = 5 trials (free throw attempts).
* The player will either make (success) or miss (failure) each shot.
* The probability of making a shot on each attempt is approximately p = 0.80.
* The outcome of each shot is independent of the outcome of any of the other shots.

1. Define the binomial random variable.

Answer: Let X be the number of successful free throw shots out of 5 shots.

1. Specify (in words) the distribution of X.

Answer: X has a binomial distribution with parameter n = 5 and p = 0.80.

1. Write the formula for the probability distribution function of X.

Answer: p(x) = P(X = x) = 5Cx⋅( 0.80 )x⋅( 0.20 )5-x for x = 0, 1, 2, 3, 4, 5 .

1. Write the probability distribution in tabular form.

|  |  |  |
| --- | --- | --- |
| **PDF of X** | |  |
| **x** | **p(x)** |  |
| 0 | 0.00032 | ← p(0) = P( X = 0 ) = 1⋅ ( 0.80 )0 ⋅ ( 0.10 )5 |
| 1 | 0.00640 | ← p(1) = P( X = 1 ) = 5⋅ ( 0.80)1 ⋅ ( 0.10 )4 |
| 2 | 0.05120 | ← p(2) = P( X = 2 ) = 10⋅ ( 0.80)2 ⋅ ( 0.10 )3 |
| 3 | 0.20480 | ← p(3) = P( X = 3 ) = 10⋅ ( 0.80)3 ⋅ ( 0.10 )2 |
| 4 | 0.40960 | ← p(4) = P( X = 4 ) = 5⋅ ( 0.80)4 ⋅ ( 0.10 )1 |
| 5 | 0.32768 | ← p(5) = P( X = 5 ) = 1⋅ ( 0.80)5 ⋅ ( 0.10 )0 |
| Total | **1.0000** |  |

1. Find the mean and the standard deviation of X. Interpret these numbers.

Answer:  = np = (5)(0.8) = 4. σ = .

In repeated sets of 5 free throw shots, the player will make on average 4 out of the 5 shots. Typically, the number of successful shots will vary from the mean of 4 by 0.894 shots.

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**Example 17**

A player wins a certain game of chance about 32% of the time. Suppose the player plays the game 6 times. Find the probability that the player wins the game exactly twice. Begin by defining the random variable X and specifying the distribution of X.

**Solution**

Let X denote the number of wins in 6 plays of the game.

X has a binomial distribution with parameters n = 6 and p = 0.32.

P(X = 2) =

**Example 18**

A fair four-sided die is tossed 8 times. Find the probability of observing no more than 1 four. Begin by defining the random variable X and specifying the distribution of X.

**Solution**

Let X denote the number of fours observed in 8 tosses of a fair four-sided die.

X has a binomial distribution with parameters n = 8 and p = 0.25.

P(X < 1) =

**Example 19**

A poorly prepared student is guessing at a 10-question multiple choice exam. Each question has 5 selections. Find the probability of observing at least 8 correct answers. Begin by defining the random variable X and specifying the distribution of X.

**Solution**

Let X denote the number of correct answers on a 10-question multiple choice exam.

X has a binomial distribution with parameters n = 10 and p = 0.20.

P(X > 8) =

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**Example 20**

Approximately 10% of the world’s population is left-handed. Consider 25 randomly selected people. Define the random variable X to be the number of lefties in the sample.

1. Specify (in words) the distribution of X.
2. Find the probability of observing no more than 4 lefties in the sample. Use the CDF feature in your calculator.
3. Find the probability of observing at least 7 lefties in the sample. Use the CDF feature in your calculator.
4. Find the probability of observing between 2 and 5 lefties in the sample. Use the CDF feature in your calculator.
5. Find the mean and standard deviation of X.